Source Modelling of Massive Black Hole Binaries

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Content:

- **Analytical modelling of MBHBs**
- **Extracting astrophysical information and testing GR**
- **Do we need more accurate waveforms which include**
  - **Spin precession**
  - **Eccentricity**
  - **Higher-order PN effects in amplitude and phase**
  - **Merger and ringdown**
- **Detection, subtraction and parameter estimation**

Massive Binary Black Holes (MBHBs)

– **SMBHB: BH-BH binaries**

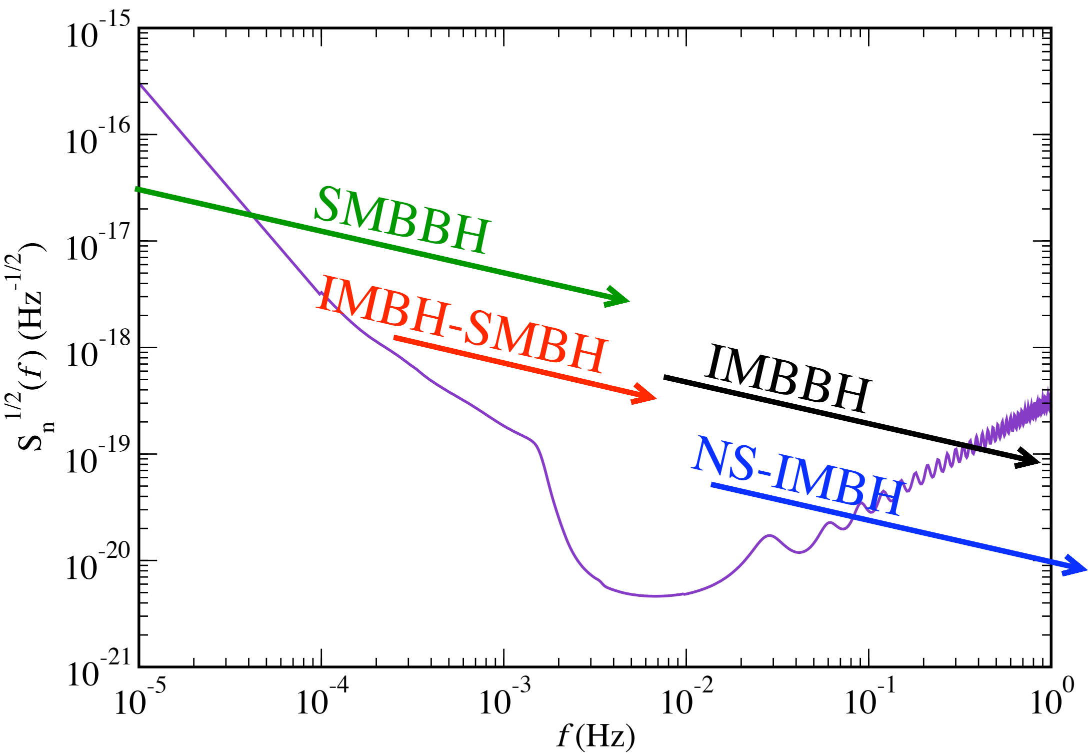
$M_{\rm BH} = 10^5$–$10^8\,M_\odot$

– **IMBH-SMBH binaries**

$M_{\rm IMBH} = 10^2$–$10^4\,M_\odot$

– **Compact body-IMBH binaries**

– **IMBHB: IMBH-IMBH binaries**



Typical features of waveforms from MBBHs

- **Inspiral: circular orbits**

  Throughout the inspiral $T_{\rm RR} \gg T_{\rm orb} \Rightarrow$ **natural** *adiabatic parameter* $\frac{\dot{\omega}}{\omega^2} = \mathcal{O}\left(\frac{v^5}{c^5}\right)$

  For compact bodies $\frac{v^2}{c^2} \sim \frac{GM}{c^2 r} \Rightarrow$ **PN approximation: slow motion and weak field**

- **Inspiral: precessing orbits**

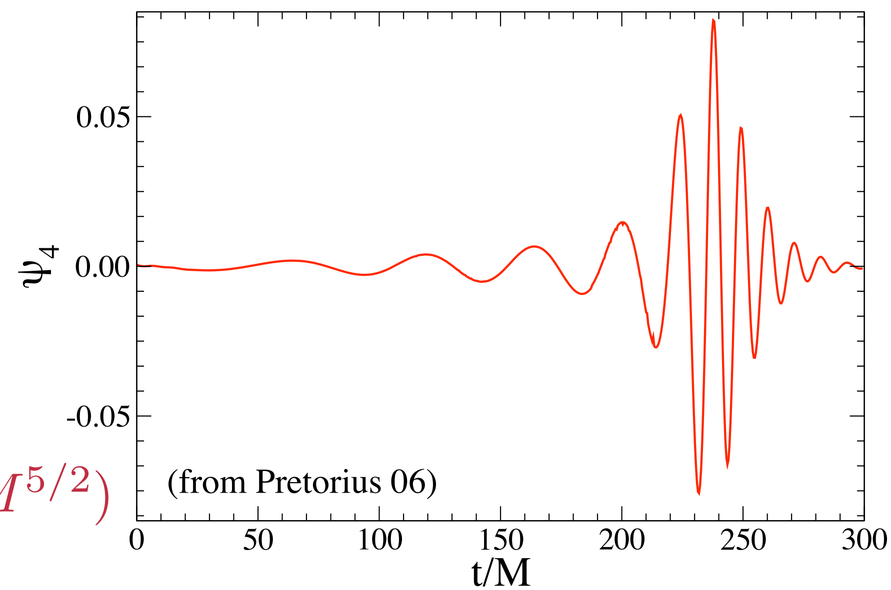
  $T_{\rm RR} \gg T_{\rm prec} \gg T_{\rm orb}$; $\omega_{\rm GW} = (\omega_{\rm prec}, 2\omega)$

- **Inspiral: eccentric orbits**

  $T_{\rm RR} \gg T_{\rm peri} \gg T_{\rm orb}$; $\omega_{\rm GW} = N\,\omega_{\rm orb}$

Chirping: $T_{\rm obs} \gtrsim \omega/\dot{\omega}$

SNR $\propto M^{5/6}\,\eta^{1/2}/D_L \quad \mathcal{N}_{\rm cycles} \sim 1/(\eta M^{5/2})$



- **Last cycles-plunge-merger-ringdown**

  Numerical relativity; **PN resummation techniques (Padé and EOB); close-limit approx.**

Extracting science from MBHB observation

- **SMBH formation** [Haehnelt; Menou et al.; Sesana et al.; Islam et al.]
  - **Large SNR; Event rates** ~ 0.1–$10^2 (10^3?)$**/year depending on z**
  - **Accuracy required: a few or tens of percent in estimating masses and distance; as highest as possible in estimating the location in the sky**
- **SMBHs as standard sirens** [Schutz; Merkovitz; Finn; Holz & Hughes ...]
  - **Accuracy required: as highest as possible in location and distance**
- **Confirming existence of IMBHBs**

  SNR ~ 10**; a few or tens/year**
- **IMBH+SMBH**

  Large SNR; a few or tens/year at $z \sim 1$
- **Tests of GR**

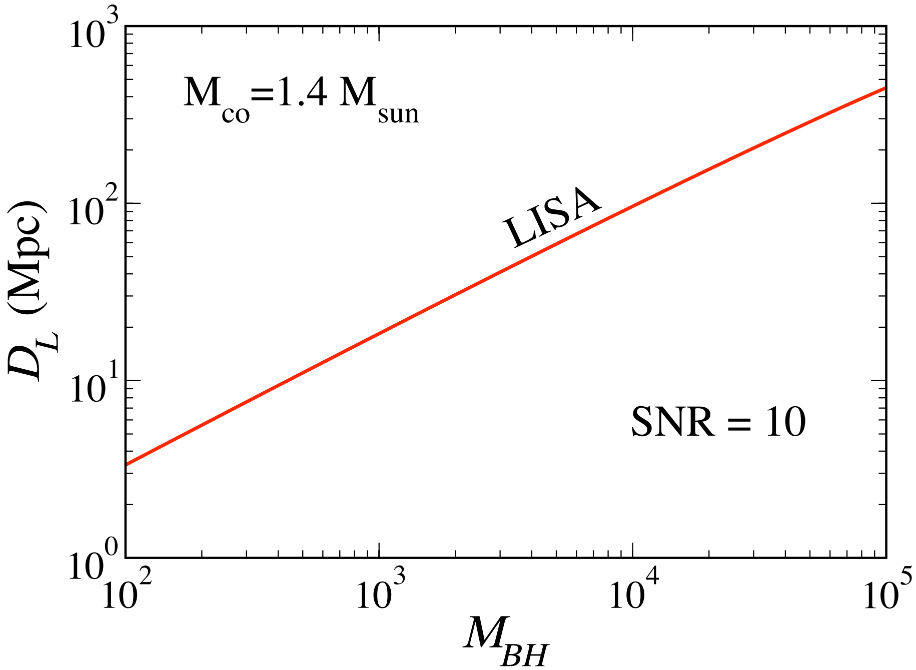
  PN approximation; non-linear/strong gravity; alternative theories; BH area theorem
  - **Accuracy required: as highest as possible in estimating** all **the binary parameters**

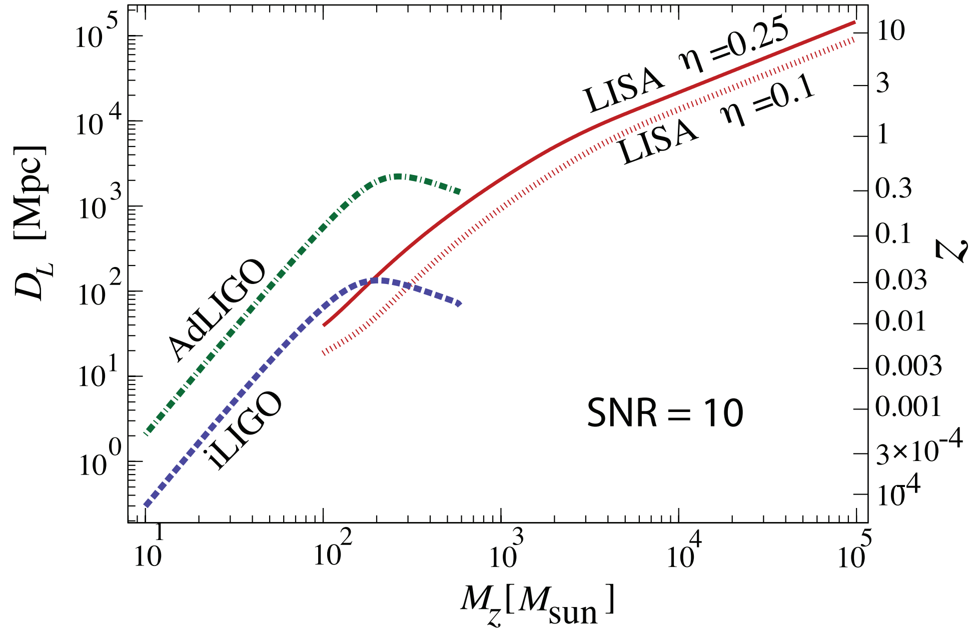
Detectability of IMBHs

- **compact body $(1\text{–}10 M_\odot)$ + IMBH**

  event rates $\sim 10^{-6}$ per year

  [Will 05]





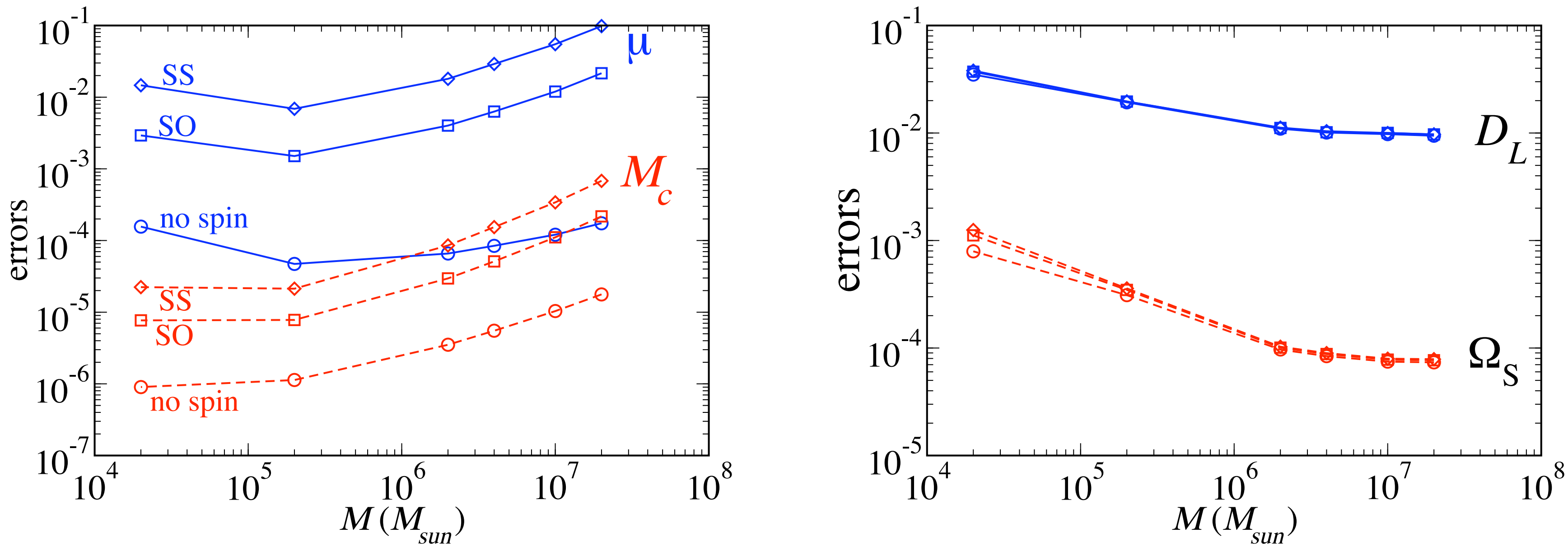
- **IMBH-IMBH**

  [Gurkan et al. 06; Fregeau et al. 06]

Parameter estimation including spin couplings: non-precessing case

Monte Carlo with 10^4 sources distributed over sky positions and orientation

$M = (10^6 + 10^6) M_\odot$ **at** 3 **Gpc** `[Berti, AB & Will 05]`

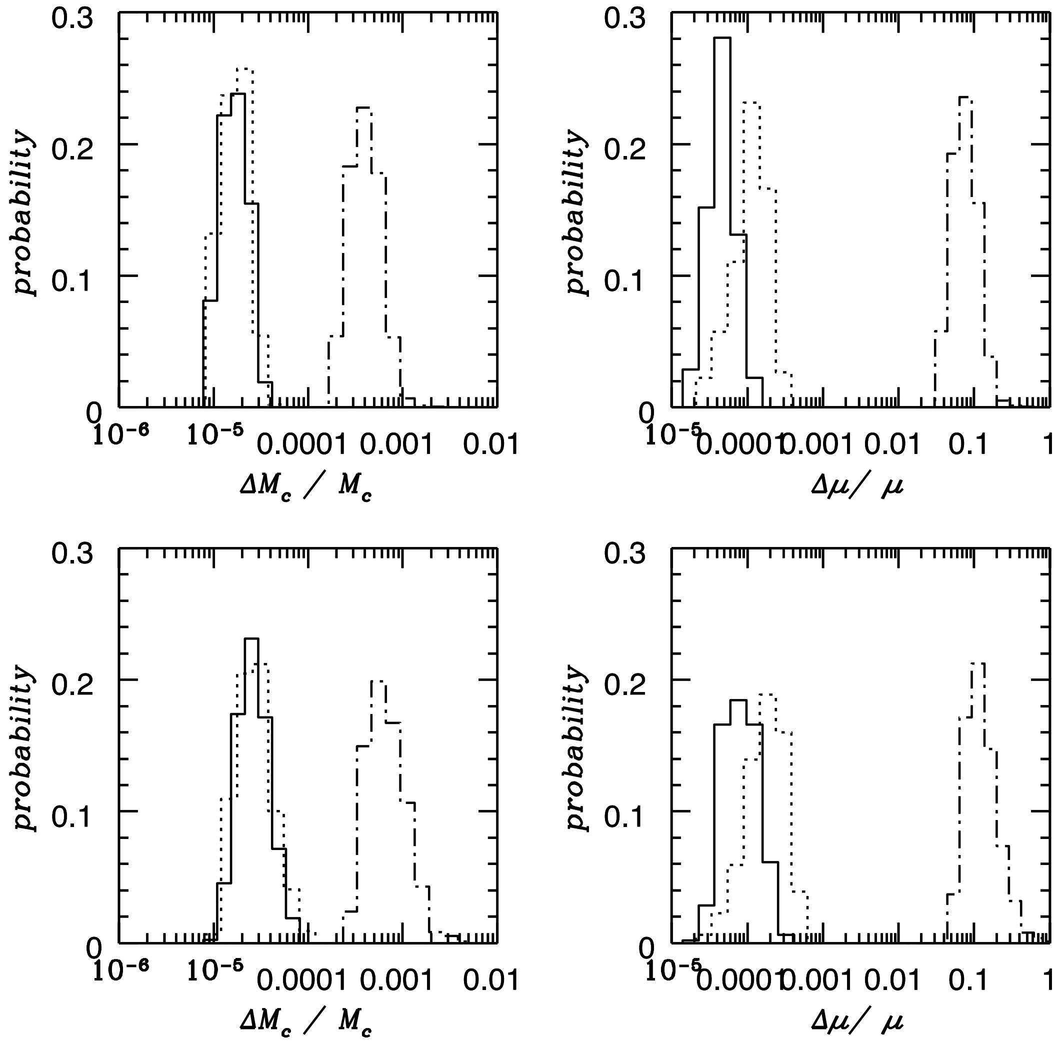


$(\overline{\phi}_S,\ \cos\overline{\theta}_S) \Rightarrow$ **binary position with respect to solar-system baricenter**

angular resolution: $\Delta\Omega_S = 2\pi \left\{ \langle \Delta\bar{\mu}_S^2 \rangle \langle \Delta\bar{\phi}_S^2 \rangle - \langle \Delta\bar{\mu}_S \Delta\bar{\phi}_S \rangle^2 \right\}^{1/2}$

$$\overline{\mu}_S = \cos\overline{\theta}_S$$

Parameter estimation including spin couplings: precessing case



[Vecchio 04]

$(10^6 + 10^6)M_\odot$

spin-orbit coupling

$\Rightarrow$ **modulations decorrelate parameters**

New study including different masses, spins, etc.

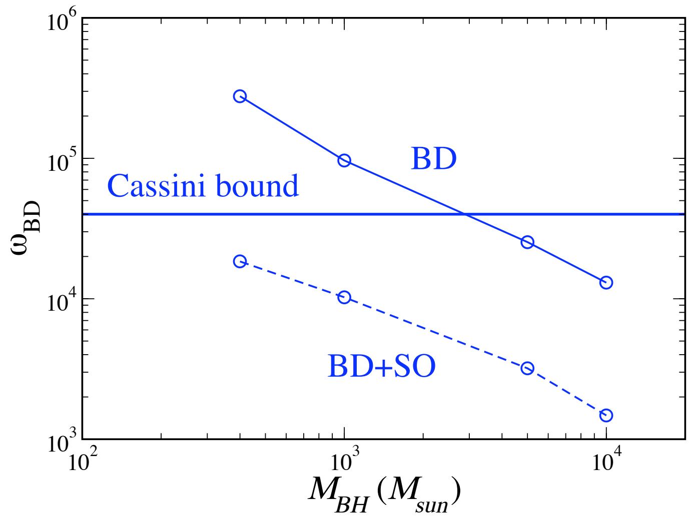
[Lang & Hughes (in preparation)]

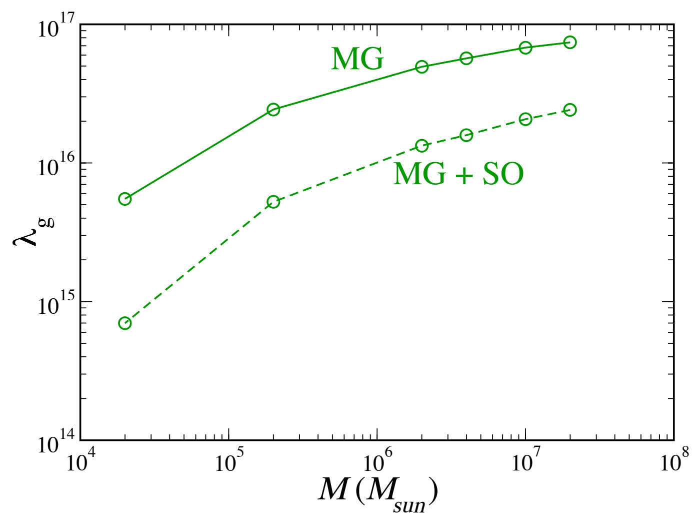
Testing Einstein general relativity

[Will 94; Krolik et al. 96; Will 98; Scharre & Will 02; Will & Yunes 04; Berti, AB & Will 05]

- **Scalar-tensor theories: phasing modified by GW dipole radiation**
- **Massive graviton theories: GW–propagation-speed depends on wavelength $\Rightarrow$ distortion in time of arrival with respect to GR**

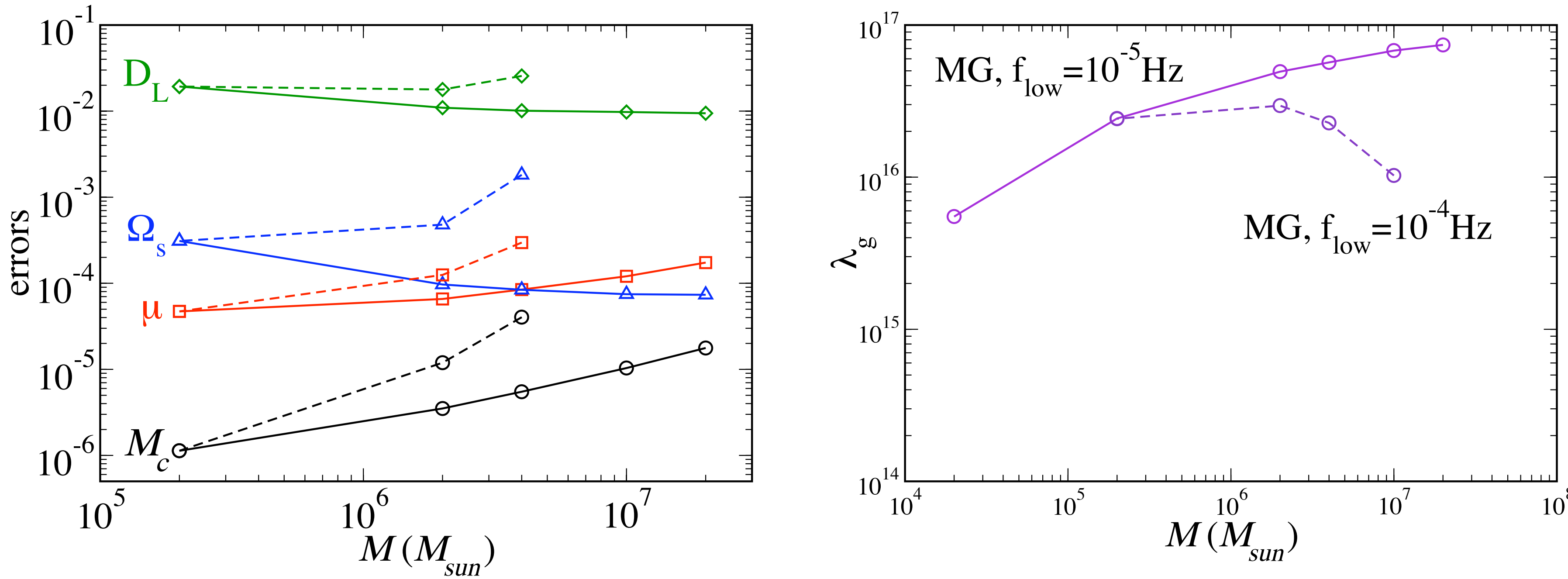
$$\dot{\omega} = \frac{96}{5\mathcal{M}^2} (\mathcal{M}\omega)^{11/3} \left\{ 1 + \frac{5\,\hat{\alpha}^2\,\eta^{2/5}}{192\,\omega_{\rm BD}} (\mathcal{M}\,\omega)^{-2/3} + \frac{96\,\pi^2\,\mathcal{M}\,D}{5\,(1+z)\,\lambda_g^2} (\mathcal{M}\omega)^{2/3} + \text{PN corr.} \right\}$$





Pushing the low-frequency cutoff at smaller frequency

$D_L = 3$ **Gpc** $\quad f_{\rm cut} = 10^{-5}$ **Hz (continuous lines)** $\quad f_{\rm cut} = 10^{-4}$ **Hz (dashed lines)**



[Berti, Buonanno & Will 05]

Effect of systematics: number of cycles (SMBHB)

$M = (10^6 + 10^6) M_\odot$ **at** 3 **Gpc**

$f_{\rm in} = 0.045{\rm m}$ Hz; $f_{\rm fin} = 2.2{\rm m}$ Hz (one year observation, SNR ~ 1861)

$\chi = |\mathbf{S}|/m^2$

	Number of cycles	**Number of** $useful$ **cycles:**
Newtonian:	2266	10
1PN:	$+134$	$+4$
1.5PN	-92	-6
Spin-orbit:	$+29\chi_1 + 29\chi_2$	$+2\chi_1 + 2\chi_2$
2PN	$+6$	$+1$
Spin-spin:	$-2\chi_1\,\chi_2$	$0.4\chi_1\,\chi_2$
2.5PN	$-9 + 8\chi_1 + 8\chi_2$	$-2 + 0.8\chi_1 + 0.8\chi_2$
3PN:	$+2$	$+1$
3.5PN:	-1	-0.5

[Blanchet, AB & Faye 06]

Effect of systematics: number of cycles (IMBHB)

$M = (10^3 + 10^3) M_\odot$ **at** 3 **Gpc**

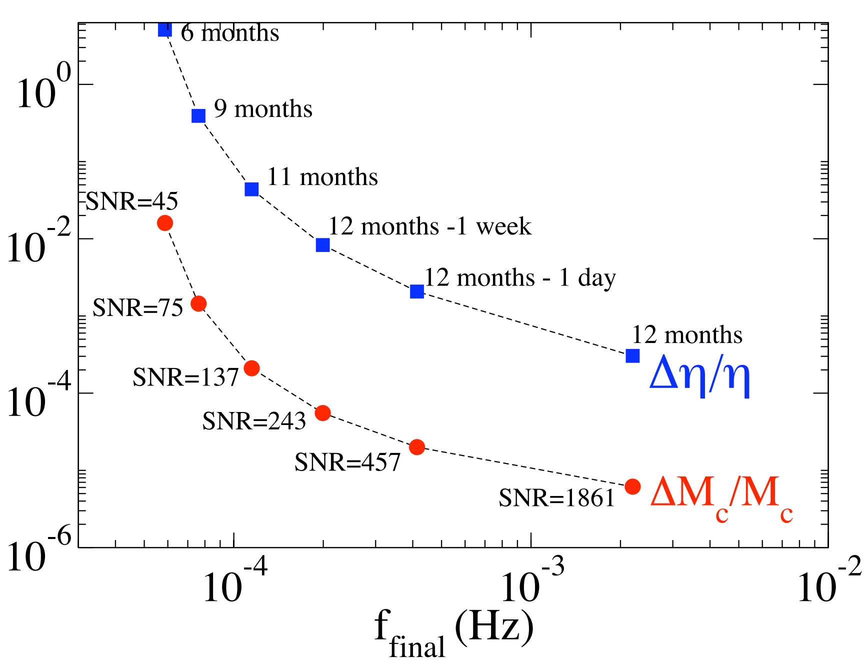
$f_{\rm in} = 3.3{\rm m}$ Hz; $f_{\rm fin} = 2$ Hz (one year observation) SNR ~ 33

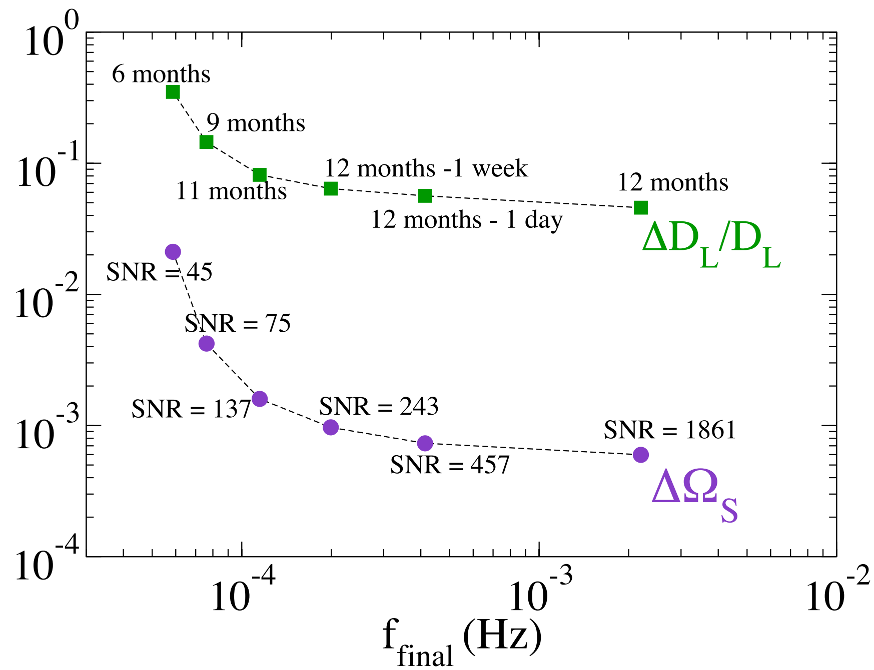
	Number of cycles	**Number of *useful* cycles:**
Newtonian:	170236	74618
1PN:	$+1828$	$+730$
1.5PN	-554	-193
Spin-orbit:	$+173\chi_1 + 173\chi_2$	$+60\chi_1 + 60\chi_2$
2PN	$+17$	$+4$
2.5PN	$-15 + 12\chi_1 + 12\chi_2$	$-2 + 0.8\chi_1 + 0.8\chi_2$
3PN:	$+3$	$+0.1$
3.5PN:	-1	-0.02

Statistical errors versus systematic errors

Monte Carlo with 10^4 sources distributed over sky positions and orientation

$M = (10^6 + 10^6)M_\odot$ **at** 3 **Gpc** [Berti, AB & Chen (work in progress)]



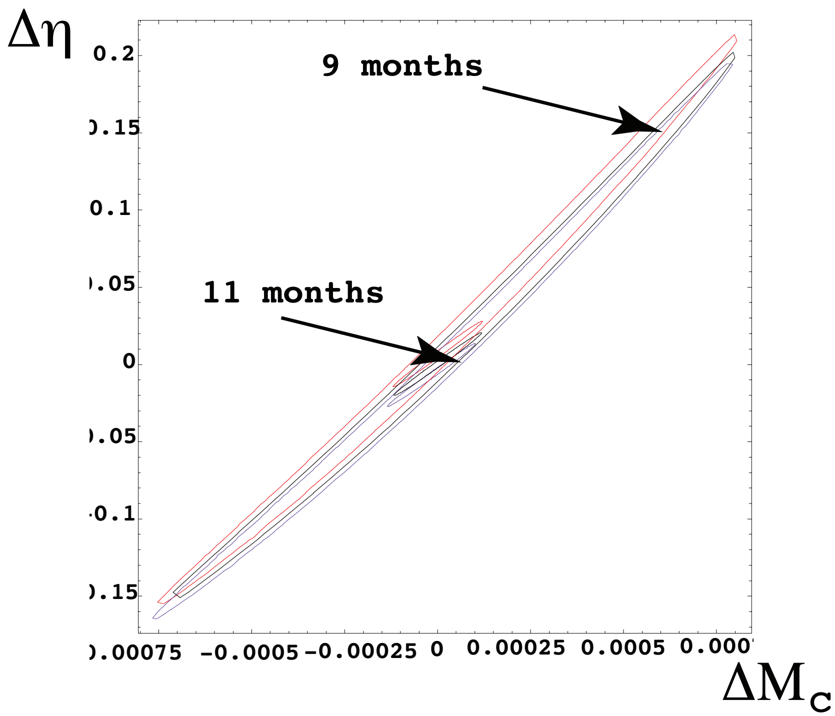


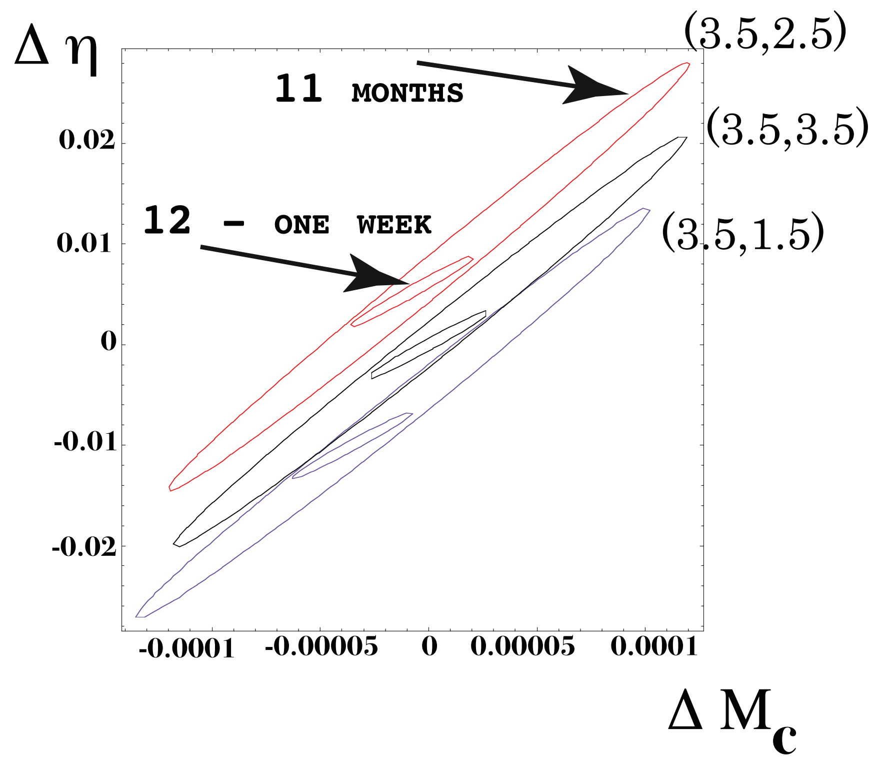
At which time are systematic errors smaller then statistical?

A possible study of statistical and systematic errors

Overlap contours at different observation times between different PN families

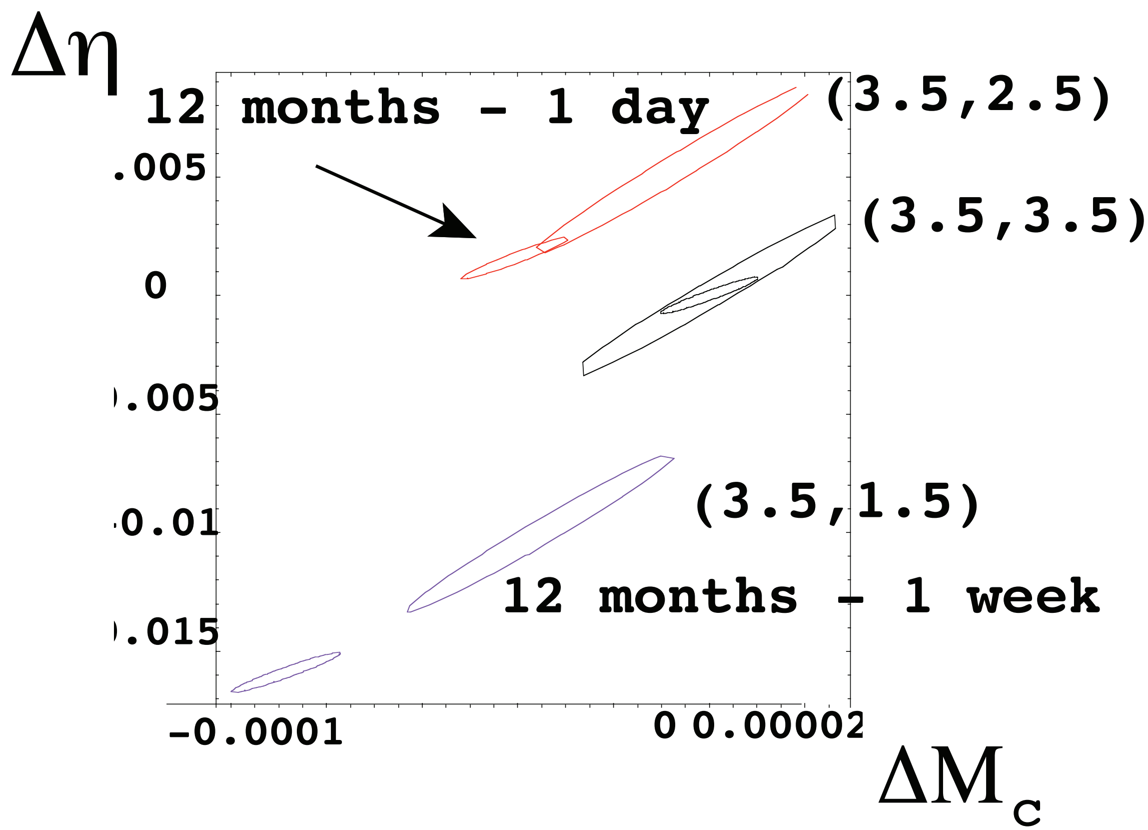
[Berti, AB & Chen (work in progress)]





Larger and larger systematics when approaching coalescence

[Berti, AB & Chen (work in progress)]

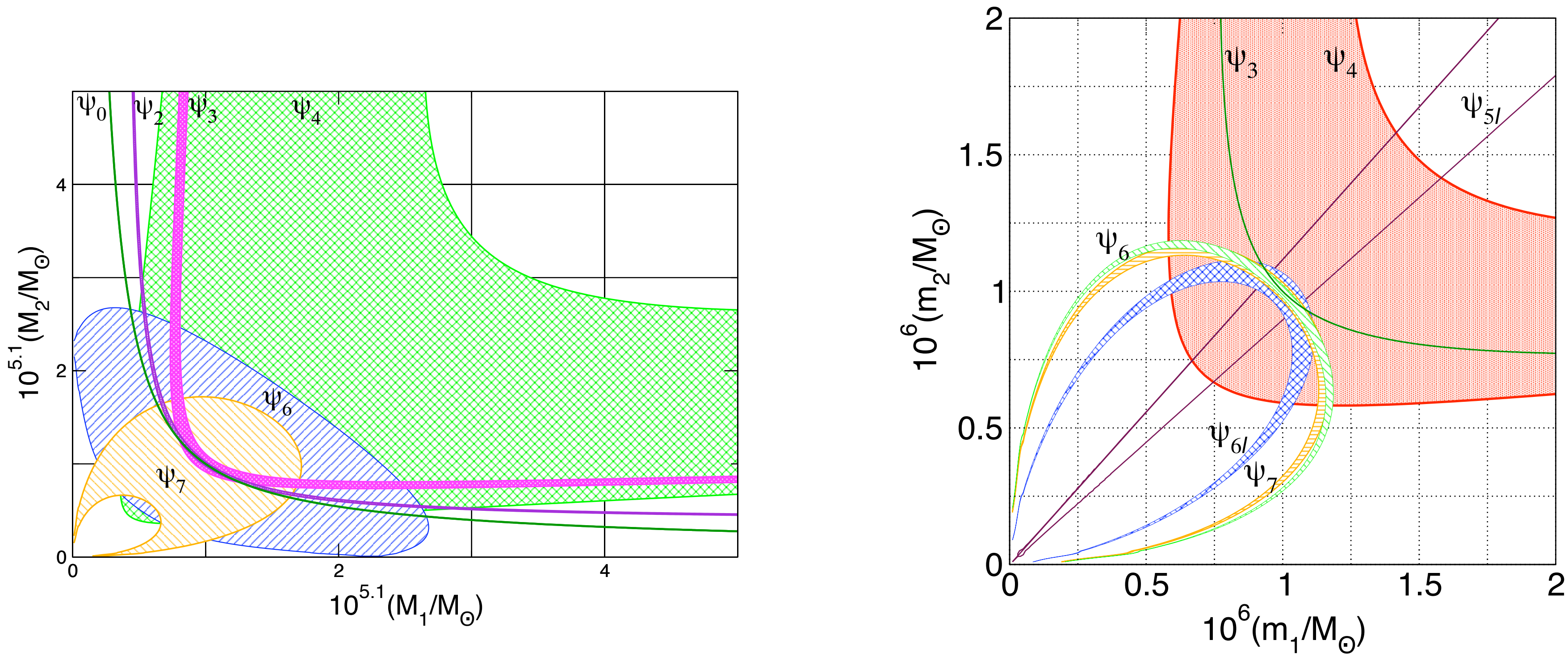


How those results change in presence of spin effects?

Testing post-Newtonian approximation

$$\Psi(f) = 2\pi\, f\, t_c - \Phi_c + \sum_{k=0}^{7} \left(\psi_k(m_1, m_2) + \psi_{k\,l}(m_1, m_2)\, \ln f\right)\, f^{(k-5)/3}$$

`[Arun, Iyer, Qusailah & Sathyaprakash 06]`

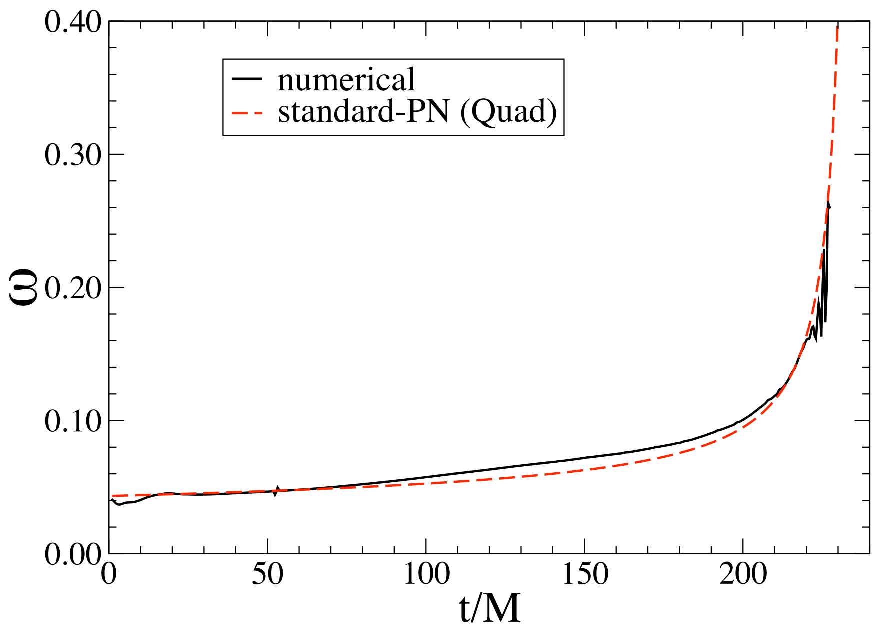


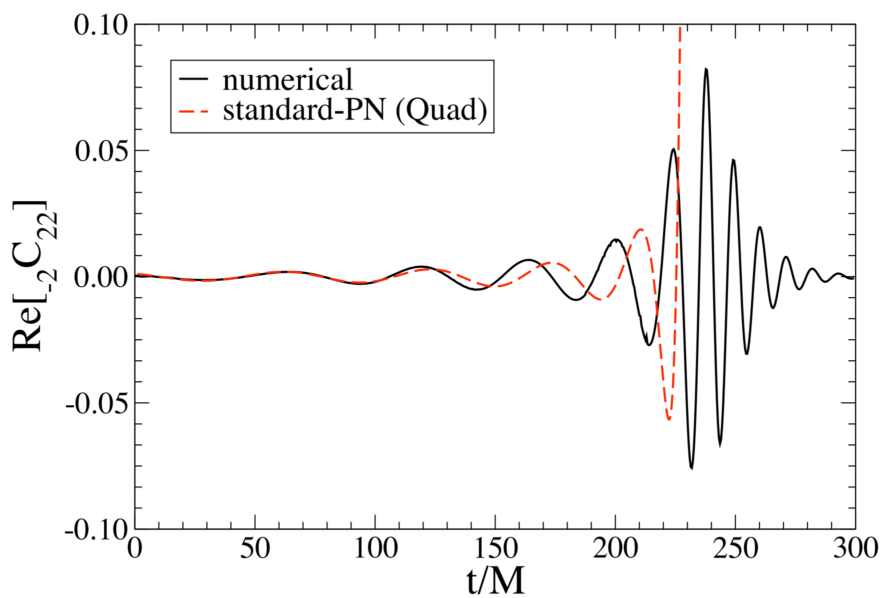
- **How those results change in presence of spins and in alternative theories of gravity?**

Preliminary comparison between quasi-circular analytical waveform and NR waveforms

[AB, Cook & Pretorius (in preparation)]

- **NR starts at $t = 0$ with $\omega = 0.0416/M$ (e.g., for a $(10^6 + 10^6)M_\odot$, $f_{\rm GW} = 1.3$ m Hz)**
  the binary evolves for 2.5 orbits





Preliminary comparison between quasi-circular analytical waveform and NR waveforms (cont.)

[AB, Cook & Pretorius (in preparation)]

- **NR starts at** $t = 0$ $\omega = 0.0416/M$ **(e.g., for a** $(10^6 + 10^6)M_\odot$**,** $f_{\rm GW} = 1.3$ **m Hz)**
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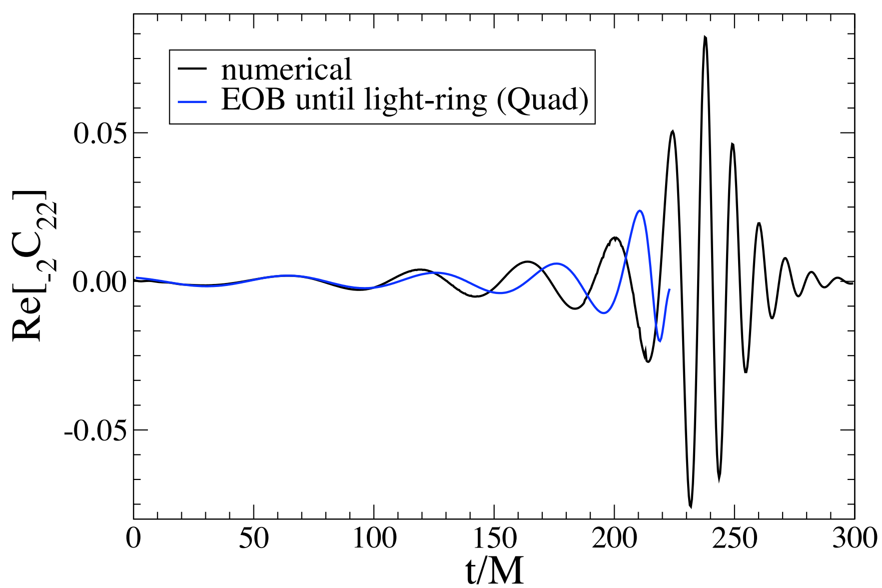
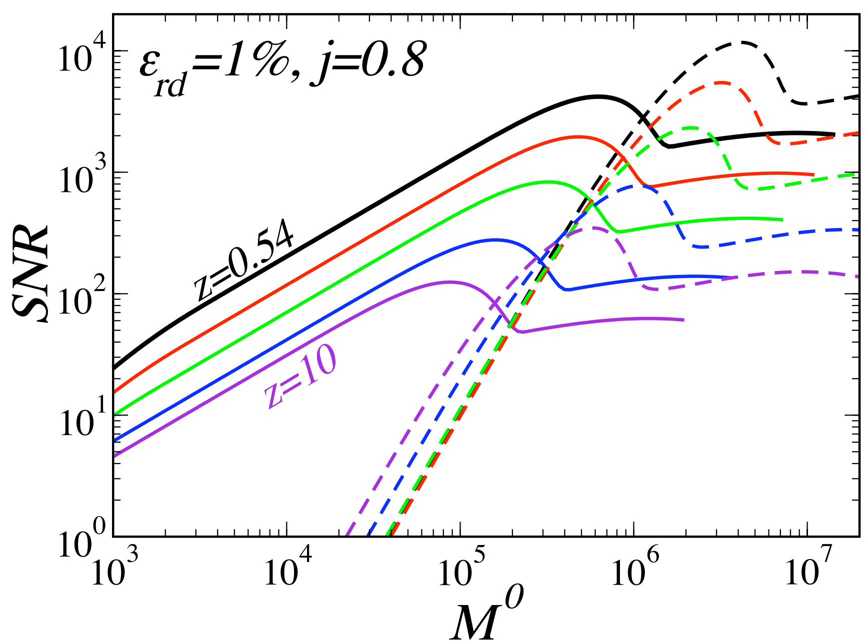


Merger and Ring down

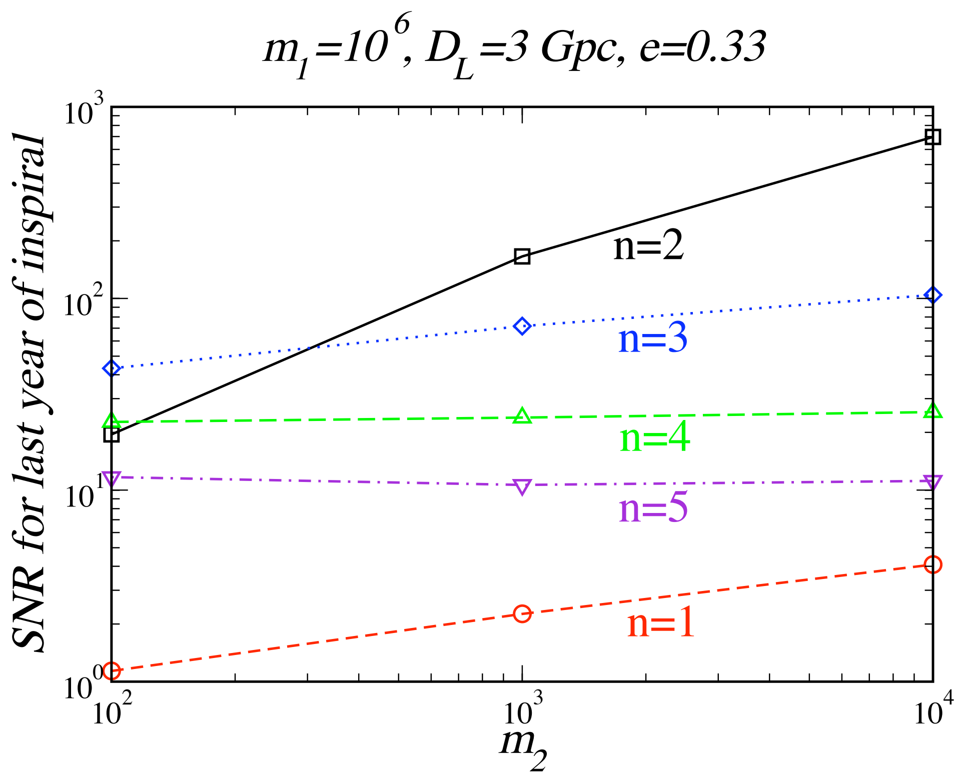
Observing high-mass BHB at larger and larger distances



[Berti, Cardoso & Will 06]

IMBH-SMBH: what about the eccentricity?

A few events per year [Miller 05; Portegies-Zwart 05, Matsubayashi et al. 05]



- **Questionable to use circular-orbit templates for detection?** **At which frequency does it circularize?**
- **Which approximation method to use for $m_2/m_1 \sim 10^{-3}$–10^{-1}?** **Combining the PN approximation with perturbation theory**

Berti, Shifflett & Will (work in progress) using waveforms from Moreno-Garrido et al. 94

Higher-order PN corrections to the signal amplitude of MBHBs

Including higher order harmonics

$$h(t) = h_1^{0.5\mathrm{PN}}\, e^{i\Phi_{\mathrm{GW}}} + h_2^{0\mathrm{PN}}\, e^{i2\Phi_{\mathrm{GW}}} + h_3^{0.5\mathrm{PN}}\, e^{i3\Phi_{\mathrm{GW}}} + \cdots$$

- **It improves estimation of binary parameters and distance, but angular resolution is almost unaffected**

- **Make it possible to observe higher-mass MBHB**

  `[Sintes & Vecchio 00; Hellings & Moore 00, 02]`

Detection, subtraction and faithfulness

- **Detection should not be a problem for SMBHBs (high SNR), unless**
  - the masses are $> 10^7 M_\odot$, thus they merge in the low-frequency band or we want to alert astronomers of a SMBHB coalescence a few months in advance
  - there are many events at the same time and we need to separate them
- **Detection/parameter estimation** $might$ **deserve more study for IMBH with SNR** ~ 10 **if we do not know sufficiently well the waveform (e.g., spin precession)**
- **Param. estim.** $might$ **deserve more study for IMBH-SMBH with mass ratio** 10^{-3}–10^{-2}
- **Which detection strategy? Matched filtering; time-frequency domain; MCMC**
- **Even if detection is not a problem, in the middle/high-frequency band of LISA we need to disantangle one signal from others with small errors**
- **Parameter estimation: warning on systematic errors**

Concluding

- **The detection of MBHBs will open an exciting new era for astronomy**
- **We need to prepare the** *best tools* **to be able to extract the** *best science* **in astrophysics, general relativity or fundamental physics**